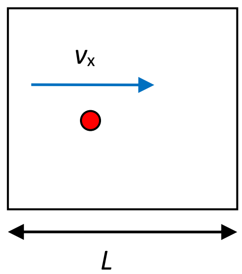
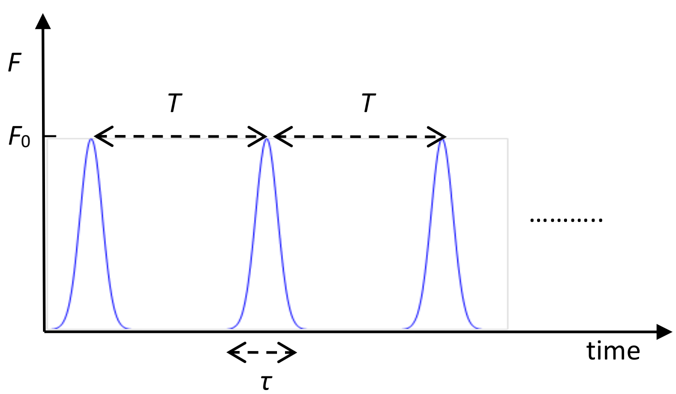
Teacher notes
Topic B

A derivation of $P = \frac{1}{3}\rho c^2$.

We assume that the molecules exert a force on the wall every time they collide with the wall.



We also assume that the force exerted varies with time as shown in the graph.



The duration of the collision is τ and the time in between collisions is T. The area under each curve is the impulse and equals J. We have that $J = 2mv_x$. The average force during each collision is $\frac{J}{\tau}$ and is impossible to calculate since we do not know τ (or v_x). So all we can do is to find the average force during a very long interval of time. We take this time to be nT where n is large. A time of nT includes n impulses and so the total impulse supplied is nJ. The average force is then $\frac{nJ}{nT} = \frac{J}{T}$. Now $T = \frac{2L}{v_x}$ and so the average force is $\frac{2mv_x}{T} = \frac{2mv_x}{\frac{2L}{v_x}} = \frac{mv_x^2}{L}$. This force acts on an area L^2 and so the pressure from one molecule is $\frac{mv_x^2}{L^3} = \frac{mv_x^2}{V}$ where V is the volume of the container. The pressure from N molecules is then

$\frac{m}{V}(v_{1x}^2+...+v_{Nx}^2)$. We define the average of the squares of the x-components of speeds to be $\overline{v_x^2}=\frac{v_{1x}^2+...+v_{Nx}^2}{N}$ so that $P=\frac{Nm}{V}\overline{v_x^2}=\frac{M}{V}\overline{v_x^2}=\rho\overline{v_x^2}$, (because *Nm* is the mass *M* of the gas and *M*/*V* is the density *ρ*.) But, by symmetry, $\overline{v_x^2}=\overline{v_y^2}=\overline{v_z^2}$ and $\overline{v_x^2}+\overline{v_y^2}+\overline{v_z^2}=c^2$ where *c* is the r.m.s. speed of the molecules, so finally $\overline{v_x^2}=\frac{1}{3}c^2$ and then $P=\frac{1}{3}\rho c^2$.